C++ Presentation 5

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Solution of system of linear equations

$$a_1 X + b_1 Y + c_1 Z = d_1$$

$$a_2 X + b_2 Y + c_2 Z = d_2$$

$$a_3 X + b_3 Y + c_3 Z = d_3$$

$$AX=B$$

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \quad \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \quad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

Methods for solving system of linear equations

1. Gauss elimination method
2. Gauss Jordan method
3. Crouts method

Direct methods
Exact solution

4. Jacobi method
5. Gauss seidel method

Indirect methods
Approx. solution

Gauss elimination method

1 AX=B

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

2 Find augmented matrix for given system. C=[A:B]

3 Transform the augmented matrix into upper triangular or echelom form

$$\begin{pmatrix} 1 & b^1{}_1 & c^1{}_1 & d^1{}_1 \\ 0 & 1 & c^1{}_2 & d^2{}_1 \\ 0 & 0 & 1 & d^3{}_1 \end{pmatrix}$$

4 Find the equations corresponding to upper triangular matrix.

5 Using back substitution, find the solution of given system of equations

Q) Solve

2X - Y + 3Z= 9

X + Y + Z= 6

X - Y + Z= 2

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \\ 2 & -1 & 3 & 9 \end{pmatrix}$$

P → (pivot: 1)

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 1 & -3 \end{pmatrix}$$

P → (pivot: -2)

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & 2 & 6 \end{pmatrix}$$

X=1
Y=2
Z=3

No of equations = n
Steps in forward elimination = n-1

(Q) Solve

$3X + 6Y + 5Z + 2W = 2$

$2X + 5Y + 2Z - 3W = 3$

$4X + 5Y + 14Z + 14W = 11$

$5X + 10Y + 8Z + 4W = 4$

Ans)

W=4

Z=6

Y=27

X=66

Drawbacks of gauss elimination technique

1.Division by zero

- $10X_2 - 7X_3 = 3$

$6X_1 + 2X_2 + 3X_3 = 11$

$5X_1 - X_2 + 5X_3 = 9$

Pivot element → (circled 0)

$$\begin{array}{r} \\ 6/0 \\ 5/0 \end{array} \begin{pmatrix} 0 & 10 & -7 \\ 6 & 2 & 3 \\ 5 & -1 & 5 \end{pmatrix}$$

- Division by zero can occur at any step of forward elimination

$12X_1 + 10X_2 - 7X_3 = 15$

$6X_1 + 5X_2 + 3X_3 = 14$

$5X_1 - X_2 + 5X_3 = 9$

$$\begin{array}{r} \\ m=6/12 \\ m=5/12 \end{array} \begin{pmatrix} 12 & 10 & -7 \\ 6 & 5 & 3 \\ 5 & -1 & 5 \end{pmatrix}$$

$$\begin{array}{r} \\ \\ m=-6.2/0 \end{array} \begin{pmatrix} 12 & 10 & -7 \\ 0 & 0 & 0.5 \\ 0 & -6.2 & 7.9 \end{pmatrix}$$

2.Large round off error

$$\begin{pmatrix} 20 & 15 & 10 \\ -3 & -2.25 & 7 \\ 5 & 1 & 3 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 45 \\ 1.751 \\ 9 \end{pmatrix} \longrightarrow \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Exact solution

- Solving it by computer upto 6 significant digits

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 0.962 \\ 1.05 \\ 0.99995 \end{pmatrix}$$

- Solving it by computer upto 5 significant digits

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 0.625 \\ 1.5 \\ 0.99995 \end{pmatrix}$$

3.Ill conditioned system of equations

- **Ill conditioned system of equations**

When there is a very small change in elements of a system, it may produce a very large changes in the solution. Such systems are said to be ill conditioned systems.

In other words a system of equations are said to be illconditioned if a small change in coefficient matrix or small change in right hand side results in large change in solution vector.

- **well conditioned system**

Show that the following system of equations is illconditioned

$$X + 5Y = 17$$

$$1.5X + 7.501Y = 25.503$$

The roots of these equations is calculated to be X=2,Y=3.

Suppose there is small change in second equations i.e

$$X + 5Y = 17$$

$$1.5X + 7.501Y = 25.5$$

The solution of this equation will be X=17,and Y=0.

Thus a very small change in coefficient bring about a change in solution from (2,3) to (17,0). These equations are then said to be illconditioned.

Solution

Increase the number of significant digits

- Decrease round off error
- Does not avoid division by zero

Gaussian elimination by partial pivoting

- Avoids division by zero
- Reduces round of error

Gauss elimination by partial pivoting

What is partial pivoting

If there are n equations===➔ n-1 steps in forward elimination

n=100 ==➔ 99 steps

Before each step of forward elimination this process of partial pivorting has to be done.

Suppose Kth step

$|a_{k\ k}|$
$|a_{k+1\ k}|$
$|a_{p\ k}|$
.
.
$|a_{n\ k}|$

maximum value is $|a_{p\ k}|$

switch row P and row K(without moving any other row)

Suppose Kth step $|a_{k\,k}|\ldots\ldots\ldots|a_{k+1\;k}|\ldots\ldots\ldots|\,a_{p\,k}|\ldots\ldots|a_{\,n\,k}|$

maximum value is $|a_{\,p\,k}|$

Rows are switched

Row 2

Row 5

$$\begin{pmatrix} 6 & 14 & 5.1 & 3.7 & 6 \\ 0 & -7 & 6 & 1 & 2 \\ 0 & 4 & 12 & 1 & 11 \\ 0 & 9 & 23 & 6 & 8 \\ 0 & -17 & 12 & 11 & 43 \end{pmatrix}$$

2 step $a_{22}, a_{32}, a_{42}, a_{52}$

Ques) Solve the following systems using Gaussian elimination using partial pivoting

$X_1 + X_2 + X_3 = 1$

$2X_1 + 3X_2 + 4X_3 = 3$

$4X_1 + 9X_2 + 16X_3 = 11$

$2X_1 + X_2 + X_3 - X_4 = -3$

$X_1 + 9X_2 + 8X_3 + 4X_4 = 15$

$-X_1 + 3X_2 + 5X_3 + 2X_4 = 10$

$X_2 + X_4 = 2$

Gauss elimination method

```
#include<iostream.h>
#define MX 20
#include<conio.h>
int main()
int n,i,j,k,sum;
int a[MX][MX+1],x[MX];
cout<<“enter the number of equations”:
cin>>n;
cout<<“enter the augmented matrix row wise”;
for(i=0;i<=n-1;i++)
{
for(j=0;j<=n;j++)
{
cin>>a[i][j];
}
```

$$\begin{array}{c} \\ i=0 \\ i=1 \\ i=2 \end{array} \begin{array}{c} \begin{array}{ccc} j=0 & j=1 & j=2 \end{array} \\ \begin{pmatrix} a_{00} & a_{01} & a_{01} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \end{array} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} \quad \begin{pmatrix} a_{03} \\ a_{13} \\ a_{23} \end{pmatrix}$$

No of equations=3

$$\begin{array}{c} \\ i=0 \\ i=1 \\ i=2 \end{array} \begin{array}{c} \begin{array}{cccc} j=0 & j=1 & j=2 & j=3 \end{array} \\ \begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \end{pmatrix} \end{array}$$

/*now we will begin Gaussian elimination*/

```
for(k=0;k<=n-2;k++)
{
for(i=k+1;i<=n-1;i++)
{
m=a[i][k]/a[k][k];

for(j=k;j<=n;j++)
a[i][j]=a[i][j]-m*a[k][j];
}}
```

Continued.....

/*READS ELEMENTS ROW WISE*/

		j=0	j=1	j=2	j=3
	k=0	a_{00}	a_{01}	a_{02}	a_{03}
i=1	k=1	a_{10}	a_{11}	a_{12}	a_{13}
i=2		a_{20}	a_{21}	a_{22}	a_{23}

3	2	4	7	R_1
5	9	5	4	R_2
2	1	6	4	R_3

$R_2 = R_2 - 5/3 R_1$

$$m = \frac{5}{3}$$

a[i][j]=a[i][j]-m*a[k][j]

$$a_{10} = a_{10} - \frac{5}{3} * a_{00}$$

$$\begin{pmatrix} 3 & 2 & 4 & 7 \\ 0 & 9 & 5 & 4 \\ 0 & 0 & 6 & 4 \end{pmatrix}$$

	j=0	j=1	j=2	j=3
i=0	a_{00}	a_{01}	a_{02}	a_{03}
i=1	0	a_{11}	a_{12}	a_{13}
i=2	0	0	a_{22}	a_{23}

$$\begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix}$$

n=3

```
X[n-1]=a[n-1][n]/a[n-1][n-1];

for(i=n-2;i>=0;i--)
{
sum=0;
for(j=i+1;j<=n-1;j++)
sum+=a[i][j]*X[j];
X[i]=(a[i][n]-sum)/a[i][i]
}
Continued.....
```

$a_{22}\ X_2 = a_{23}$ 1st equation

$X_2 = a_{23} / a_{22}$

$a_{11}\ X_1 + a_{12} X_2 = a_{13}$ 2nd equation

$\text{Sum} = a_{12} X_2$

$X_1 = (a_{13} - \text{sum})/a_{11}$

```
for(i=0;i<n;i++)
{
cout<<X[i];
}
End….
```

Complete pivoting

$$X_1 - X_2 = 0$$
$$-X_1 + 2X_2 - X_3 = 1$$
$$-X_2 + 4X_3 = 0$$

$$\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$R_1 \longleftrightarrow R_3$

$$\left(\begin{array}{ccc|c} X_1 & X_2 & X_3 & \\ 0 & -1 & 4 & 0 \\ -1 & 2 & -1 & 1 \\ 1 & -1 & 0 & 0 \end{array}\right)$$

$C_1 \longleftrightarrow C_3$ ➡

$$\left(\begin{array}{ccc|c} X_3 & X_2 & X_1 & \\ 4 & -1 & 0 & 0 \\ -1 & 2 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right)$$

$R_1 \rightarrow R_2 - (-1/4)R_1$

$$\left(\begin{array}{ccc|c} X_3 & X_2 & X_1 & \\ 4 & -1 & 0 & 0 \\ 0 & 7/4 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right)$$

$R_3 \rightarrow R_3 - (-4/7)R_2$

$$\left(\begin{array}{ccc|c} X_3 & X_2 & X_1 & \\ 4 & -1 & 0 & 0 \\ 0 & 7/4 & -1 & 1 \\ 0 & 0 & 3/7 & 4/7 \end{array}\right)$$

$$\begin{pmatrix} X_3 \\ X_2 \\ X_1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 4/3 \\ 4/3 \end{pmatrix}$$

$$X_1 + X_2 + X_3 = 1$$
$$2X_1 + 3X_2 + 4X_3 = 3$$
$$4X_1 + 9X_2 + 16X_3 = 11$$

X_1	X_2	X_3	
1	1	1	1
2	3	4	3
4	9	16	11

$R_1 \longleftrightarrow R_3$

X_1	X_2	X_3	
4	9	16	11
2	3	4	3
1	1	1	1

$C_1 \longleftrightarrow C_3$

X_3	X_2	X_1	
16	9	4	11
4	3	2	3
1	1	1	1

X_3	X_2	X_1	
16	9	4	11
0	3/4	1	1/4
0	7/16	3/4	5/16

$R_2 \rightarrow R_2 - (4/16)R_1$

$R_3 \rightarrow R_3 - (1/16)R_1$

X_3	X_1	X_2	
16	4	9	11
0	1	3/4	1/4
0	3/4	7/16	5/16

$R_3 \rightarrow R_3 - (3/4)R_2$

$C_2 \longleftrightarrow C_3$

X_3	X_1	X_2	
16	4	9	11
0	1	3/4	1/4
0	0	-1/8	1/8

Solution: $X_3 = 1, X_1 = 1, X_2 = -1$

SOLVE

$$2X_1 + X_2 + X_3 - X_4 = -3$$

$$X_1 + 9X_2 + 8X_3 + 4X_4 = 15$$

$$-X_1 + 3X_2 + 5X_3 + 2X_4 = 10$$

$$X_2 + X_4 = 2$$

$X_1 = -1$,

$X_2 = 0$,

$X_3 = 1$,

$X_4 = 2$

Gauss jordan method

1 AX=B

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

2 Find augmented matrix for given system. C=[A:B]

3 Transform the augmented matrix into normal echelom form

$$\begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{pmatrix}$$

4 Find the solution directly.

$X + 3Y + 3Z = 16$

$X + 4Y + 3Z = 18$

$X + 3Y + 4Z = 19$

$$\begin{pmatrix} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 3 & 19 \end{pmatrix}$$

$$\begin{pmatrix} \textcircled{1} & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 3 & 10 \\ 0 & \textcircled{1} & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & \textcircled{1} & 3 \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Solve

$$6X - Y + Z = 13$$
$$X + Y + Z = 9$$
$$10X + Y - Z = 19$$

$$C = \begin{pmatrix} 1 & 1 & 1 & 9 \\ 6 & -1 & 1 & 13 \\ 10 & 1 & -1 & 19 \end{pmatrix}$$

$R_2 = R_2 - 6R_1$
$R_3 = R_3 - 10R_1$

$$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & -7 & -5 & -41 \\ 0 & -9 & -11 & -71 \end{pmatrix}$$

$R_3 = R_3 - R_2$
$R_3 \leftrightarrow R_2$

$$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & -2 & -6 & -30 \\ 0 & -7 & -5 & -41 \end{pmatrix}$$

$R_2 = R_2/(-2)$

$$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & 3 & 15 \\ 0 & -7 & -5 & -41 \end{pmatrix}$$

$R_3 = R_3 + 7R_2$

$$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & 16 & 64 \end{pmatrix}$$

$R_3 = R_3 / 16$

$$\begin{pmatrix} 1 & 0 & 1 & 9 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

$R_1 = R_1 - R_3$

$$\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{pmatrix} \quad R_1 = R_1 - R_2 \quad \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

$X=2$

$Y=3$

$Z=4$

Q)

$2X_1+2X_2-X_3+X_4=4$

$4X_1+3X_2-X_3+2X_4=6$

$8X_1+5X_2-3X_3+4X_4=12$

$3X_1+3X_2-2X_3+2X_4=16$

Solution

$X_1=1$

$X_2=1$

$X_3=-1$

$X_4=-1$

Q)

$X_1+X_2+X_3+4X_4=-6$

$X_1+7X_2+X_3+X_4=12$

$X_1+X_2+6X_3+X_4=-5$

$5X_1+X_2+X_3+X_4=4$

Solution

$X_1=1$

$X_2=2$

$X_3=-1$

$X_4=-2$

Q)

$2X_1+4X_2-6X_3=-8$

$X_1+3X_2+X_3=10$

$2X_1-4X_2-2X_3=-12$

Q)

$X_1+X_2+2X_3=1$

$2X_1-X_2+X_4=-2$

X1-X2-X3-$2X_4=4$

$2X_1-X_2+2X_3-X_4=0$

Gauss Jordan method program

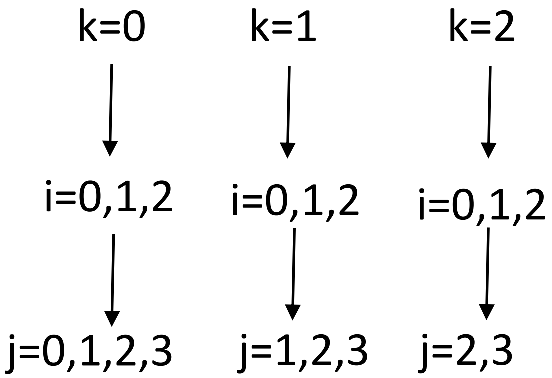
```
#include<iostream.h>
#define MX 20
#include<conio.h>
int main()
int n,i,j,k,sum;
cout<<"enter the number of equations":
cin>>n;
int a[MX][MX+1],x[MX];
cout<<"enter the augmented matrix row wise";
for(i=0;i<=n-1;i++)
{
for(j=0;j<=n;j++)
{
cin>>a[i][j];
}
continued.....
```

$$\begin{array}{c} \\ i=0 \\ i=1 \\ i=2 \end{array} \begin{array}{ccc} j=0 & j=1 & j=2 \\ \end{array} \begin{pmatrix} a_{00} & a_{01} & a_{01} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} \begin{pmatrix} a_{03} \\ a_{13} \\ a_{23} \end{pmatrix}$$

No of equations=3

$$\begin{array}{c} i=0 \\ i=1 \\ i=2 \end{array} \begin{pmatrix} j=0 & j=1 & j=2 & j=3 \\ a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \end{pmatrix}$$

```
for(k=0;k<=n-1;k++) //k=0,1,2
{
for(i=0;i<=n-1;i++)   //i=0,1,2
{
m=a[i][k]/a[k][k];
dummy_value=a[k][k];
for(j=k;j<=n;j++)
{
if(i==k)
a[i][j]=a[i][j]/dummy_value;
else
a[i][j]=a[i][j]-m*a[k][j];
}}}
for(i=0;i<=n-1;i++)
cout<<a[i][n];
}  /*end*/
```



j=0,1,2,3 for (k=0)

j=1,2,3 for (k=1)

j=2,3 for (k=2)

		j=0	j=1	j=2	j=3
k=0	i=0	a_{00}	a_{01}	a_{02}	a_{03}
k=1	i=1	a_{10}	a_{11}	a_{12}	a_{13}
k=2	i=2	a_{20}	a_{21}	a_{22}	a_{23}

/*explanation*/

K=0, i=0

j=0,1,2,3 **dummy_value=2**

a[0][0]=a[0][0]/dummy_value

a[0][1]=a[0][1]/dummy_value

a[0][2]=a[0][2]/dummy_value

a[0][3]=a[0][3]/dummy_value

STEP I

$R_1 \rightarrow R_1/1^{ST}$

j=0 j=1 j=2 j=3

$$\begin{pmatrix} 2 & 1 & 3 & 4 \\ 5 & 2 & 3 & 7 \\ 2 & 4 & 1 & 5 \end{pmatrix}$$

j=0 j=1 j=2 j=3

i=0

$$\begin{pmatrix} 1 & 0.5 & 1.5 & 2 \\ 5 & 2 & 3 & 7 \\ 2 & 4 & 1 & 5 \end{pmatrix}$$

$R_2 \rightarrow R_2-(5/1)R_1$

k=0

↓

i=0,1,2

↓

j=0,1,2,3

K=0,i=1 **m=5/1**

STEP I

$R_2 \rightarrow R_2-m*R_1$

a[i][j]=a[i][j]-m*a[k][j]

j=0 $a_{10} = a_{10} - \frac{5}{1} * a_{00}$

j=1 $a_{11} = a_{11} - (5/1)* a_{01}$

j=2 $a_{12} = a_{12} - (5/1)* a_{02}$

j=3 $a_{13} = a_{13} - (5/1)* a_{03}$

i=1

j=0 j=1 j=2 j=3

$$\begin{pmatrix} 1 & 0.5 & 1.5 & 2 \\ 0 & -0.5 & 4 & -3 \\ 2 & 4 & 1 & 5 \end{pmatrix}$$

STEP I

$R_3 \rightarrow R_3-(2/1)R_1$

m=2/1

k=0,i=2

j=0

j=1

j=2

j=3

j=0 j=1 j=2 j=3

$$\begin{pmatrix} 1 & 0.5 & 1.5 & 2 \\ 0 & -0.5 & 4 & -3 \\ 0 & -1 & -8 & -9 \end{pmatrix}$$

k=1,i=0 /*explanation*/

j=1 $a_{01} = a_{01} - m*a_{11}$

j=2 $a_{02} = a_{02} - m*a_{12}$

j=3 $a_{03} = a_{03} - m*a_{13}$

K=1, i=1 **a[i][j]=a[i][j]/dummy_value**

j=1 $a_{11}=a_{11}$/dummy_value

j=2 $a_{12}=a_{12}$/dummy_value

j=3 $a_{13}=a_{13}$/dummy_value

dummy_value=-0.5

k=1, i=2 **a[i][j]=a[i][j]-m*a[k][j]**

m=3/1

j=1 $a_{21}=a_{21}-m*a_{11}$

j=2 $a_{22}=a_{22}-m*a_{12}$

j=3 $a_{23}=a_{23}-m*a_{13}$

j=0	j=1	j=2	j=3
1	0.5	1.5	2
0	-0.5	-4	-3
0	-1	-8	-9

STEP II

$R_1 \rightarrow R_1 - m*R_2$

m=-0.5/0.5

j=0	j=1	j=2	j=3
1	0	2.5	-1
0	1	8	6
0	3	-2	1

i=1

STEP II

$R_2 \rightarrow R_2/2^{nd}$

j=0	j=1	j=2	j=3
1	0	2.5	-1
0	1	8	6
0	0	-26	-17

i=0

STEP II

$R_3 \rightarrow R_3 - m*R_2$

m=3/1

k=1 → i=0,1,2 → j=1,2,3

k=2,i=0

j=2

j=3

j=0	j=1	j=2	j=3
1	0	0	-0.25
0	1	8	6
0	0	-2	1

$R_1 \rightarrow R_1 - mR_3$

k=2

↓

i=0,1,2

↓

j=2,3

K=2, i=1

j=2

j=3

j=0	j=1	j=2	j=3
1	0	0	-0.25
0	1	0	10
0	0	-2	1

$R_2 \rightarrow R_2 - mR_3$

k=2, i=2

j=2

j=3

j=0	j=1	j=2	j=3
1	0	0	-0.25
0	1	0	10
0	0	1	-0.5

$R_3 \rightarrow R_3 / -2$

Methods for solving system of linear equations

1. Gauss elimination method
2. Gauss Jordan method
3. Crouts method
4. Jacobi method
5. Gauss seidel method

Direct methods
Exact solution

Indirect/Iterative method
Approx. solution

ITERATIVE METHODS

Let us consider the given system of equation

$$a_{11}X_1+a_{12}X_2+a_{13}X_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{1n}X_n=C_1$$

$$a_{21}X_1+a_{22}X_2+a_{23}X_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{2n}X_n=C_2$$

$$a_{31}X_1+a_{32}X_2+a_{33}X_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{3n}X_n=C_3$$

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$$a_{n1}X_1+a_{n2}X_2+a_{n3}X_3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{nn}X_n=C_n$$

System is rewritten as

$X_1=1/a_{11}\ \ (C_1-a_{12}X_2-a_{13}X_3-\ldots\ldots\ldots\ldots\ldots\ldots a_{1n}X_n)$

$X_2=1/a_{22}\ \ (C_2-a_{21}X_1-a_{23}X_3-a_{13}X_3-\ldots\ldots\ldots\ldots\ldots\ldots a_{2n}X_n)$

$X_3=1/a_{33}\ \ (C_3-a_{31}X_2-a_{32}X_2-\ldots\ldots\ldots\ldots\ldots\ldots a_{3n}X_n)$

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$X_n=1/a_{nn}\ \ (C_n-a_{n1}X_1-a_{n2}X_2-\ldots\ldots\ldots\ldots\ldots\ldots a_{n(n-1)}X_{n-1})$

NOTE: for the Gauss-seidel method to converge quickly the coefficient matrix should be diagonally dominant.
Coefficient matrix is said to be diagonally dominant if the numerical values of leading diagonal elements in each row is greater than or equal to sum of numerical values of other elements in the row.

$$\begin{pmatrix} 5 & 1 & -1 \\ 1 & 4 & 2 \\ 2 & -2 & 5 \end{pmatrix}$$

diagonally dominant

$$\begin{pmatrix} 5 & 1 & -1 \\ 5 & 2 & 3 \\ 1 & -2 & 5 \end{pmatrix}$$

NOT diagonally dominant

Solve by Gauss-seidel method

X-2Y=-3---(1)

2X+25Y=15---(2)

Solve by Gauss-seidel method

X-2Y=-3--(1)

2X+25Y=15--(2)

Solution:

X=-3+2Y--(3)

Y=1/25(15-2X)--(4)

1st Iteration

Putting Y=0 we get X=-3

Putting X=-3 in equ(4) we get Y=1/25(15-2(-3))=0.84

X=-3,Y=0.84

2nd Iteration

Putting Y=0.84 in equ(3) we get X=-3+2(0.84)=-1.32

Putting X=-1.32 in equ(4) we get Y=1/25(15-2(-1.32))=0.7056

X=-1.32,Y=0.7056

3rd Iteration

Putting Y=0.7056 in equ(3) we get X=-1.589

Putting X=-1.589 in equ(4) we get Y=1/25(15-2(-1.589))=0.727

X=-1.589,Y=0.727

2nd Iteration

Putting Y=0.727 in equ(3) we get X=-3+2(0.727)=-1.546

Putting X=-1.546 in equ(4) we get Y=1/25(15-2(-1.546))=0.724

X=-1.546,Y=0.724

4th Iteration

Putting Y=0.724 in equ(3) we get X=-1.552

Putting X=-1.552 in equ(4) we get Y=1/25(15-2(-1.552))=0.724

X=-1.552,Y=0.724

5th Iteration

Putting Y=0.724 in equ(3) we get X=-3+2(0.724)=-1.552

Putting X=-1.552 in equ(4) we get Y=1/25(15-2(-1.552))=0.724

X=-1.552,Y=0.724

RESULT: X=-1.552,Y=0.724

(Q)Solve the system of equations

$$4X+2Y+Z=14$$

$$X+5Y-Z=10$$

$$X+Y+8Z=20$$

NOTE: Check whether the coefficient matrix is diagonally dominant

Solution : X=2.001,Y=1.999,Z=2

(Q)Solve the system of equations

$$3X-Y+Z=1$$

$$3X+6Y+2Z=0$$

$$3X+3Y+7Z=4$$

Solution : X=0.0351,Y=-0.2368,Z=0.6578

(Q) Solve the system of equations by Gauss seidel method

$$X + Y + 54Z = 110$$

$$27X + 6Y - Z = 85$$

$$6X + 15Y + 2Z = 72$$

Solution: X=2.42547, Y=3.5730, Z=1.92595

ERRORS AND APPROXIMATION

- Limitation of analytical methods lead to numerical methods.
- Even if analytical solution is available ,these are not amenable to direct numerical interpretation.
- With increasing demand for numerical solution to various problems, numerical techniques have become indispensable tools for engineers and scientist.
- Error in final result may be due to error in initial data or the method or both.

Accuracy of numbers

1. Appropriate numbers

There are two types of numbers exact and appropriate

- Exact : which can be expressed by finite number of digits.(2,4,7/2,9/3)

There are numbers which cant be expressed by finite number of digits.

4/3=1.33333…….; π=3.141592……; sqrt(2)=1.414213…………

These may be approximated by 1.3333,1.4142

- Approximate: are numbers which represent the given numbers to certain degree of accuracy.

2. Significant digits

The digits used to express a number are called significant digits(figures).

7845=>4 significant digits

3.589=>4 significant digits

0.000386=> 3 significant digits

45000=>2 significant digits

3. Rounding off

There are numbers with large digits i.e 22/7=3.142857143 .in practice it is desirable to limit such numbers to a manageable number of digits such as 3.14 or 3.143. This process of dropping unwanted digits is called rounding off.

Rule to round off to nth significant digits

If the discarded number is

- Less than half a unit in nth place ,leave the nth digit unchanged
- Greater than half a unit in nth place ,increase the nth digit by unity
- Exactly half a unit in nth place, increase the nth digit by unity if it is odd otherwise leave it unchanged.

Sources of errors

1. Inherent errors: errors which are already present in the statement of a problem before its solution are called as inherent errors. Such errors arise due to the given data being approximate or due to limitations of mathematical tables, calculators or digital computers.

These can be minimised by taking better data, using high precision instruments.

2. Round off errors: arise from the process of rounding off the numbers during the computation. Such errors are unavoidable in many cases due to the limitation of computing aids.

(a) By changing the calculation procedure so as to avoid subtraction of nearly equal numbers or division by a smaller number;

(b) By retaining at least one more significant figure at each step than that given in the data and rounding off at last step.

3. Truncation errors: are caused by approximate results or on replacing an infinite process by a finite one.

$e^x=1+x+(x^2/2!)+(x^3/3!)+.......=X(say)$

$1+x+(x^2/2!)+(x^3/3!)=X^*$

Truncation error=$X-X^*$

4. Absolute error, relative error and percentage error

X true value

X^*approx. value

$E_a= I\ X-X^*\ I$

$E_r= I\ (X-X^*)/X\ I$

$E_p=100*\ I\ (X-X^*\ )/X\ I$

(Q) Round off the numbers 865250 and 37.46235 to four significant figures and compute Ea, Er, Ep in each case.

Gauss seidel concept

$$6X_0 - 2X_1 + X_2 = 11$$
$$X_0 + 2X_1 - 5X_2 = -1$$
$$-2X_0 + 7X_1 + 2X_2 = 5$$

$$X_0 = (11 + 2X_1 - X_2) / 6$$
$$X_1 = (-1 - X_0 + 5X_2) / 2$$
$$X_2 = (5 + 2X_0 - 7X_1) / 2$$

GAUSS SEIDEL METHOD

```
#include<iostream.h>
#define MX 20
#include<math.h>
int main()
{
int n,i,j,k,flag=0;
cout<<"enter no of equations":
cin>>n;
float a[MX][MX+1],X[MX],eps,y;
cout<<"enter the elements of augmented matrix:";
for(i=0;i<n;i++)
for(j=0;j<=n;j++)
cin>>a[i][j];
```

```
/*initialisation*/
cout<<"enter the initial values of variables";

for(i=0;i<n;i++)
cin>>X[i];

cout<<"enter the accuracy upto which you want the solution";

cin>>eps;
/*continued*/
```

```
for(i=0;i<n;i++)
{
y=X[i];
X[i]=a[i][n];
for(j=0;j<n;j++)
{
if(j!=i)
X[i]=X[i]-a[i][j]*X[j];
}
X[i]=X[i]/a[i][i];

If(abs(X[i]-y)<=eps)
flag++;
cout<<X[i];
}/*continued*/
```

i=0	i=1	i=2
j=0	j=1	j=2
j=1	j=1	j=1
j=2	j=2	j=2

$6X_0 \quad -2X_1 \quad +X_2 \quad = 11$

$X_0 \quad +2X_1 \quad -5X_2 \quad = -1$

$-2X_0 \quad +7X_1 \quad +2X_2 \quad = 5$

$X_0 = (11 + 2X_1 - X_2)/6$

$X_1 = (-1 - X_0 + 5X_2)/2$

$X_2 = (5 + 2X_0 - 7X_1)/2$

	j=0	j=1	j=2	j=3
i=0	a_{00}	a_{01}	a_{02}	a_{03}
i=1	a_{10}	a_{11}	a_{12}	a_{13}
i=2	a_{20}	a_{21}	a_{22}	a_{23}

$$\begin{pmatrix} 6 & -2 & 1 & 11 \\ 1 & 2 & -5 & -1 \\ -2 & 7 & 2 & 5 \end{pmatrix}$$

i=0

$Y=X[0]=X_0=a[0][3]=11$

1st j=0 Skip

2nd j=1

X[0]=X[0]-a[0][1]*X[1]

$X_0=X_0-a_{01}*X_1$

$X_0=X_0-(-2)X_1=11+2X_1$

3rd j=2

$X[0]=X[0]-a[0][2]*X_2$

$X[0]=11+2X_1-a_{02}*X_2$

$X_0=11+2X_1-X_2$

X[0]=X[0]/a[0][0]

$X_0=(11+2X_1-X_2)/6$

i=1

$Y=X[1]=X_1=a[1][3]=-1$

1st j=0

X[1]=X[1]-a[1][0]*X[0]

$X_1=X_1-a_{10}*X_0$

$X_1= -1-X_0$

2nd j=1 Skip

3rd j=2

$X[1]=X[1]-a[1][2]*X_2$

$X[1]= -1-X_0 -a_{12}*X_2$

$X_1=-1-X_0+5X_2$

X[1]=X[1]/a[1][1]

$X_1=(-1-X_0+5X_2)/2$

i=2

$Y=X[2]=X_2=a[2][3]=5$

1st j=0

X[2]=X[2]-a[2][0]*X[0]

$X_2=X_2-a_{20}*X_0$

$X_2= 5-(-2)X_0 = 5+2X_0$

2nd j=1

$X[2]=X[2]-a[2][1]*X_1$

$X[2]= 5+2X_0 -a_{21}*X_1$

$X_2=5+2X_0-7X_2$

3rd j=2 Skip

X[2]=X[2]/a[2][2]

$X_1=(5+2X_0-7X_2)/2$

```
while(flag<n);
cout<<“the solution is as follows”;
for(i=0;i<n;i++)
cout<<X[i];
}
/*end*/
```

LAGRANGES INTERPOLATION

```
#include<iostream.h>
#include<conio.h>
#define MAX 10
void main()
{
int n,i,j;
float X[MAX+1],f[MAX+1];
float XX,fx,prod;
cout<<“give the order of polynomial needed”;
cin>>n;
cout<<“tabulate the value of X and f(x)”;
for(i=0;i<=n;i++)
cin>>X[i]>>f[i];
cout<<“give the value of X where f(x) is computed”;
cin>>XX;
fx=0;
```

Suppose the given polynomial is quadratic
i.e degree=2; i.e n=2
Since n=2, n+1 data points required

n=2

X	0	1	2
f(X)	0	2	8

Suppose to find f(2.5)=?
Therefore XX=2.5 f(XX)=?

```
for(i=0;i<=n;i++)
{
prod=1;
for(j=0;j<=n;j++)
{
if(j!=i)
prod*=(XX-X[j])/(X[i]-X[j])   ;
}
fx+=f[i]*prod;
}
cout<<"the values of f(X) ="<<fx;
getch();
}
```

i=0
prod=1
1st **j=0** **Skip**
2nd **j=1**
prod=1*$(XX-X_1)/(X_0-X_1)$
3rd **j=2**
prod=$(XX-X_1)/(X_0-X_1)*(XX-X_2)/(X_0-X_2)$
fx=$f(0)*(XX-X_1)/(X_0-X_1)*(XX-X_2)/(X_0-X_2)$

i=0	i=1	i=2
j=0	j=1	j=2
j=1	j=1	j=1
j=2	j=2	j=2

i=1
prod=1
1st **j=0** prod=$(XX-X_0)/(X_1-X_0)$
2nd **j=1** skip
3rd **j=2**
prod=$(XX-X_0)/(X_1-X_0)*(XX-X_2)/(X_1-X_2)$
fx=$f(0)*(XX-X_1)/(X_0-X_1)*(XX-X_2)/(X_0-X_2)+f(1)*(XX-X_0)/(X_1-X_0)*(XX-X_2)/(X_1-X_2)$

i=2
prod=1
1st **j=0** prod=$(XX-X_0)/(X_2-X_0)$
2nd **j=1** prod=$(XX-X_0)/(X_2-X_0)*(XX-X_1)/(X_2-X_1)$
3rd **j=2** **skip**

fx=$f(0)*(XX-X_1)/(X_0-X_1)*(XX-X_2)/(X_0-X_2)+f(1)*(XX-X_0)/(X_1-X_0)*(XX-X_2)/(X_1-X_2)$
$+ f(2)\ )*(XX-X_0)/(X_2-X_0)*(XX-X_0)/(X_2-X_1)$